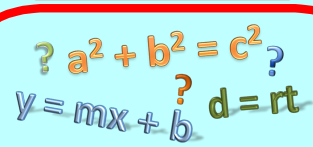
Math 115

Spring 2019

Lecture 17



Exponential Rules:

① $x^n = \underbrace{x \cdot x \cdot x \cdots x}_{n \text{ times}}$

② $x^1 = x$

③ $x^0 = 1, \quad x \neq 0$

④ $x^m \cdot x^n = x^{m+n}$

⑤ $(x^m)^n = x^{m \cdot n}$

⑥ $(xy)^n = x^n y^n$

⑦ $\frac{x^m}{x^n} = x^{m-n}$

⑧ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

⑨ $x^{-n} = \frac{1}{x^n}$

⑩ $\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$

⑪ $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

Simplify:

① $2020^{1} - 2019^{0} = 2020 - 1 = \boxed{2019}$

② $\left(\frac{2}{3}\right)^{-1} + \left(\frac{4}{5}\right)^{-1} = \left(\frac{3}{2}\right)^{1} + \left(\frac{5}{4}\right)^{1}$

$= \frac{3}{2} + \frac{5}{4} = \frac{3\cdot 2}{2\cdot 2} + \frac{5}{4} = \frac{6}{4} + \frac{5}{4}$

$= \boxed{\frac{11}{4}}$

$= \boxed{2\frac{3}{4}}$

③ $x^{7} \cdot x^{3} = x^{7+3}$

$= \boxed{x^{10}}$

④ $(x^{7})^{3}$

$= x^{7\cdot 3}$

$= \boxed{x^{21}}$

⑤ $(-4x^{5})^{3}$

$= (-4)^{3}(x^{5})^{3} = \boxed{-64x^{15}}$

⑥ $\frac{x^{12}}{(x^{5})^{2}} = \frac{x^{12}}{x^{10}}$

$= x^{12-10} = \boxed{x^{2}}$

⑦ $(x^{4})^{-3} = x^{-12}$

$= \boxed{\frac{1}{x^{12}}}$

⑧ $\left(\frac{2x^{3}}{5y^{2}}\right)^{3}$

$= \frac{(2x^{3})^{3}}{(5y^{2})^{3}} = \frac{2^{3}(x^{3})^{3}}{5^{3}(y^{2})^{3}}$

$= \boxed{\frac{8x^{9}}{125y^{6}}}$

⑨ $x^{\frac{2}{3}} \cdot x^{\frac{1}{5}}$

$= x^{\frac{2}{3}+\frac{1}{5}} = x^{\frac{10+3}{15}} = \boxed{x^{\frac{13}{15}}}$

⑩ $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}}$

$= x^{\frac{3}{4}-\frac{1}{2}} = x^{\frac{3-2}{4}} = \boxed{x^{\frac{1}{4}}}$

⑪ $\left(\dfrac{x^4 y^{-3}}{x^{-5} y^2}\right)^6$

$= \left(\dfrac{x^4\, x^5}{y^3\, y^2}\right)^6 = \left(\dfrac{x^9}{y^5}\right)^6$

$= \dfrac{(x^9)^6}{(y^5)^6} = \boxed{\dfrac{x^{54}}{y^{30}}}$

⑫ $\left(\dfrac{2x^5}{y^8}\right)^{-2}$

$= \left(\dfrac{y^8}{2x^5}\right)^2$

$= \dfrac{(y^8)^2}{(2x^5)^2} = \boxed{\dfrac{y^{16}}{4x^{10}}}$

⑬ $\left(x^{\frac{3}{4}}\right)^{\frac{2}{9}}$

$= x^{\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \cdot \frac{\cancel{2}}{\cancel{9}_{3}}} = \boxed{x^{\frac{1}{6}}}$

⑭ $\dfrac{\overset{8}{\cancel{32}}\, x^8 y^3}{\cancel{12}\, x^2 y^{10}}$

$= \boxed{\dfrac{8\ x^6}{3\ y^7}}$

⑮ $\left(\dfrac{-3x^{-4}}{2y^{-8}}\right)^{-3}$

$= \left(\dfrac{-3\ y^8}{2\ x^4}\right)^{-3} = \left(\dfrac{2x^4}{-3y^8}\right)^3 = \dfrac{2^3(x^4)^3}{(-3)^3(y^8)^3} = \boxed{\dfrac{-8x^{12}}{27y^{24}}}$

Simplify:

1) $3(4x^2 - 5x + 8) - 6(2x^2 + 5x + 4)$

$= \cancel{12x^2} - 15x \cancel{+24} \cancel{-12x^2} - 30x \cancel{-24} = \boxed{-45x}$

2) $10x^4 \cancel{-13x^3} + 10x^2 - 15x \cancel{-8} - 8x^4 \cancel{+13x^3}$
$- 15x^2 + 10x \cancel{+8}$

$= \boxed{2x^4 - 5x^2 - 5x}$

Evatuate $\dfrac{x^2 - 9}{x + 3}$ for

a) $x = 0$

$\dfrac{0^2 - 9}{0 + 3} = \dfrac{-9}{3} = \boxed{-3}$

b) $x = 3$

$\dfrac{3^2 - 9}{3 + 3} = \dfrac{9 - 9}{6} = \dfrac{0}{6}$

$= \boxed{0}$

c) $x = -3$

$\dfrac{(-3)^2 - 9}{-3 + 3} = \dfrac{9 - 9}{-3 + 3} = \dfrac{0}{0}$

Indeterminate

d) $x = 4$

$\dfrac{4^2 - 9}{4 + 3} = \dfrac{16 - 9}{7} = \dfrac{7}{7}$

$= \boxed{1}$

Monomial = Number · Variable(s) whole # exponent

$5x^3$, $-12x^2y^4$, $\frac{2}{3}xyz$, $-\frac{1}{2}xy^2z^3$

whenever monomial has no Variable, it is called constant.

1, -45, 2019, $\frac{5}{7}$, $-\frac{3}{8}$

The exponent or sum of exponents in a monomial is called degree.

The number in front of the variables in a monomial is called Coefficient.

$23x^5$
Deg. = 5
Coef. = 23

$-12x^4y^3$
Deg. = 4+3 = 7
Coef. = −12

$\frac{2}{3}xy^2z$
Deg. = 1+2+1 = 4
Coef. = $\frac{2}{3}$

Degree of constant monomial is 0.

2020
Monomial
Constant
Deg. = 0

$\frac{-45}{91}$
Monomial
Constant
Deg. = 0

Find degree and Coef.:

a) $-x^5$
Deg. = 5
Coef. = −1

b) x^6y^8z
Deg. = 6+8+1 = 15
Coef = 1

c) 90210
Constant
Deg. = 0

Binomial : when we have + or − of two monomials.

$3x+5$, x^2-10x , $x+y$, $\frac{2}{3}x^2-\frac{1}{4}y^3$

highest degree = Deg. of binomial

Coef. of the monomial with highest degree is called Leading Coef.

$\underbrace{3x}_{D=1,\ C=3} + \underbrace{5}_{D=0}$ → Deg = 1, L.C. = 3; 5 → Constant

$\underbrace{4x^2}_{D=2,\ C=4} - \underbrace{13x}_{D=1,\ C=-13}$ → D=2, L.C.=4

$\underbrace{-23x^2}_{D=2,\ C=-23} + \underbrace{y^8}_{D=8,\ C=1}$ → D=8, L.C.=1

Give deg., coef of each term, then deg. & Coef. of binomial

a) $x^4 + 1000$

x^4: D=4, C=1; 1000: Constant, D=0

$\underbrace{\quad}_{D=4,\ L.C.=1}$

b) $-x^3y^5 + 28x^2y^4$

$-x^3y^5$: D=8, C=−1; $28x^2y^4$: D=6, C=28

$\underbrace{\quad}_{D=8,\ L.C.=-1}$

c) $-x^5y^4z + 1000x^{12}z^8$

$-x^5y^4z$: D=10, C=−1; $1000x^{12}z^8$: D=20, C=1000

$\underbrace{\quad}_{D=20,\ LC=1000}$

Trinomials: when we + or − 3 monomials

$3x^2 - 5x + 10$ Trinomial

D=2,

L.C.=3

Constant=10

$-7x^3y^4 + 12xy^3 - 100x^2y$

Trinomial

D=7, L.C.=−7, No Constant

$x^6 - 25x^4y - y^5$

Trinomial, D=6, L.C.=1, No constant

Give deg, Coef, and name:

1) 1000 Constant, Monomial, D=0

2) $-8x^5y^7$ Monomial

D=12

C=−8

3) $23x^6 - 400x^2$

Binomial

D=6, L.C.=23

No constant

4) $\frac{2}{3}x^{10}y^8 - \frac{1}{2}x^7y^5 + 2019$

Trinomial, D=18, L.C.=$\frac{2}{3}$, Constant=2019

Polynomial: when we + or − monomials.

Monomial, binomial, Trinomial are all Polynomial.

$12x^5 - 8x^4 + 27x^3 - 16x^2 - 13x + 100$

Polynomial, D=5, L.C.=12, Constant =100

$\underbrace{-4x^6y^5}_{D=11} \underbrace{+13x^8y^7}_{D=15} \underbrace{-23x^{12}y^6}_{D=18} \underbrace{+x^3y^4}_{D=7} \underbrace{-2020}_{D=0}$

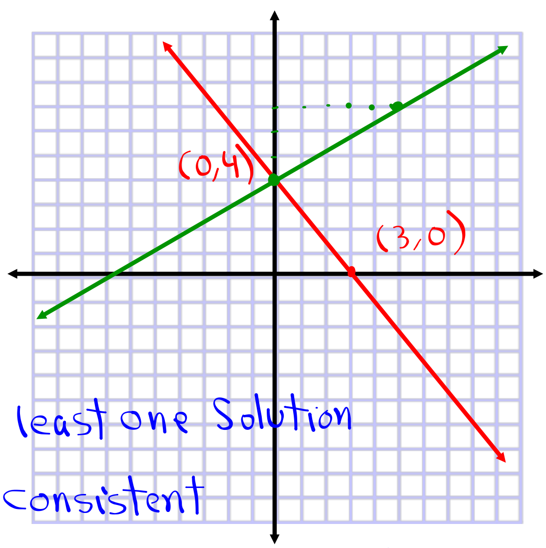
Rewrite in descending order.

$-23x^{12}y^6 + 13x^8y^7 - 4x^6y^5 + x^3y^4 - 2020$

Polynomial, D=18, L.C.=-23, Constant=-2020

Solve by graphing

$$\begin{cases} 4x + 3y = 12 \\ y = \frac{3}{5}x + 4 \end{cases}$$



Soln (0,4)

when we have at least one Solution
System is consistent

when we have exactly one Solution
Equations are independent.

Solve by Subs. method:

$$\begin{cases} y = 3x - 5 \\ 6x - 2y = 10 \end{cases}$$

$6x - 2(3x - 5) = 10$

$\cancel{6x} - \cancel{6x} + 10 = 10$ → $10 = 10$

True

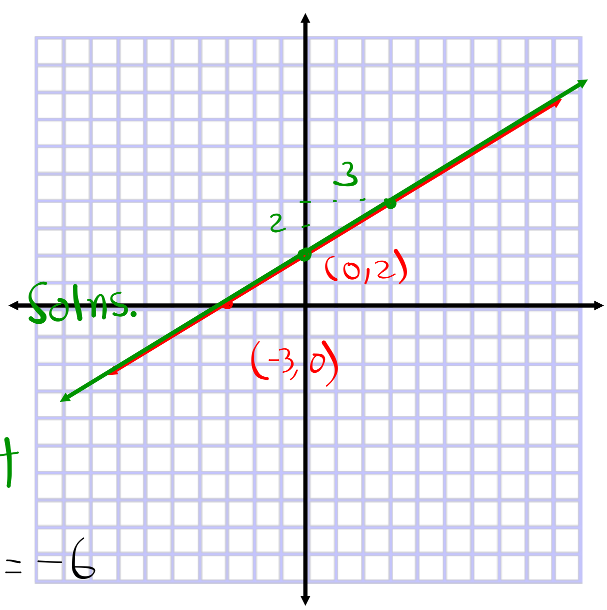
infinite # of Solutions

The system is consistent because there is at least one solution.

The equations are dependent because there are infinitely many solutions.

Solve by graphing

$$\begin{cases} 2x - 3y = -6 \\ y = \frac{2}{3}x + 2 \end{cases}$$



Infinitely Many Solns.

System: Constistent

Eqns : Dependent

$2x - 3\left(\frac{2}{3}x + 2\right) = -6$

$2x - \cancel{3} \cdot \frac{2}{\cancel{3}}x - 3 \cdot 2 = -6$

$\cancel{2x} - \cancel{2x} - 6 = -6$ → $-6 = -6$

True

Solve by addition elimination

$$-2\begin{cases} 3x - 2y = 5 \\ 6x - 4y = -10 \end{cases} \Rightarrow \begin{cases} \cancel{-6x} + \cancel{4y} = -10 \\ \cancel{6x} - \cancel{4y} = -10 \end{cases}$$

$$0 = -20$$

False

NO Solution

$\emptyset$

System : Inconsistent

Equations : Independent

Solve by graphing

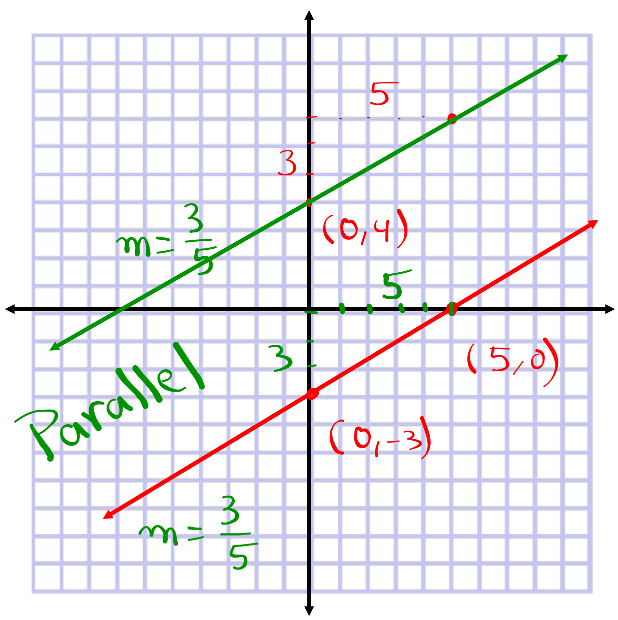
$$\begin{cases} y = \frac{3}{5}x + 4 \\ 3x - 5y = 15 \end{cases}$$

NO Solution

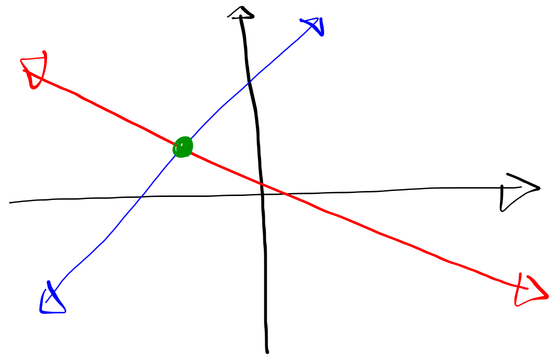
System : inconsistent

Eqns : Independent

Parallel



Lisa did a problem by graphing method:



1) what quadrant is the solution located?

QII

2) system consistent or inconsistent?

3) are equations dependent or independent?

John solved a system of linear equations, and got $-12 = 12$.

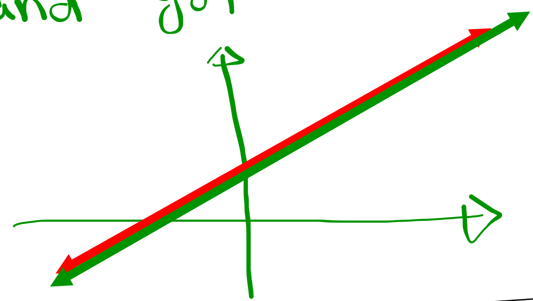
→ False

1) what is the solution? → No Solution

2) system: Inconsistent

3) Equations: Independent.

Maria Solved a system of linear equations and got $5=5$, then she tried graphing and got



True → infinitely many Solns

1) Solution

2) System Consistent

3) Equations Dependent.

1) Quiz at 6 on this topic. Score will be added to exam 2.

2) Work on SG 13, bring it to class.